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Formula Sheet for Probability and Applied Statistics

Definition 1.1

The mean of a sample of n measured responses , , . . . , is given by

The corresponding population mean is denoted μ.

Standard Deviation 1.2

The variance of a sample of measurements , , . . . , is the sum of the square of the differences between the measurements and their mean, divided by n − 1. Symbolically, the sample variance is

The corresponding population variance is denoted by the symbol σ2.

Definition 1.3

The standard deviation of a sample of measurements is the positive square root

of the variance; that is,

The corresponding population standard deviation is denoted by σ =

Definition 2.1

An experiment is the process by which an observation is made.

Definition 2.2

A simple event is an event that cannot be decomposed. Each simple event corresponds to one and only one sample point. The letter E with a subscript will be used to denote a simple event or the corresponding sample point.

Definition 2.3

The sample space associated with an experiment is the set consisting of all possible sample points. A sample space will be denoted by S.

Definition 2.4

A discrete sample space is one that contains either a finite or a countable number of distinct sample points.

Definition 2.5

An event in a discrete sample space S is a collection of sample points—that is, any subset of S.

Definition 2.6

Suppose S is a sample space associated with an experiment. To every event A in S (A is a subset of S), we assign a number, P(A), called the probability of A, so that the following axioms hold:

Axiom 1:

Axiom 2:

Axiom 3: If , *,* …. Form a sequence of pairwise mutually exclusive events in S (that is, if ), then

,) =

Theorem 2. 1

With elements , ..., and n elements , ,..., , it is possible to form pairs containing one element from each group.

Verification of the theorem can be seen by observing the rectangular table in Figure 2.9. There is one square in the table for each , pair and hence a total of squares.

Definition 2.7

An ordered arrangement of distinct objects is called a *permutation*. The number of ways of ordering distinct objects taken at a time will be designated by the symbol .

Theorem 2.2 Permutations

We are concerned with the number of ways of filling positions with distinct objects. Applying the extension of the rule, we see that the first object can be chosen in one of ways. After the first is chosen, the second can be chosen in(−1) ways, the third in (−2), and the th in (−r +1) ways. Hence, the total number of distinct arrangements is

Expressed in terms of factorials,

Where and

Theorem 2.3 Multinomial coefficients

Definition 2.8

The number of combinations of objects taken at a time is the number of

subsets, each of size , that can be formed from the objects. This number will

be denoted by or ()

Theorem 2.4 Combinations

Definition 2.9

Conditional Probability: The *conditional probability of an event A*, given that an event *B* has occurred, is equal to

Definition 2.10

A and B are Independent if

Theorem 2.5

The Multiplicative Law of Probability:

The probability of the intersection of two events A and B is

If A and B are independent, then

Theorem 2.6

The Additive Law of probability:

The probability of the union of two events A and B is

If A and B are mutually exclusive events,

Theorem 2.7

If A is an event, then

Theorem 2.8

Total Probability:

Theorem 2.9

Bayes’ Rule:

Definition 2.13

Let and represent the numbers of elements in the population and sample, respectively. If the sampling is conducted in such a way that each of the () samples has an equal probability of being selected, the sampling is said to be random, and the result is said to be a *random sample.*

Definition 3.2

The probability that Y takes on the value y, defined as the *sum of the probabilities of all sample points* in S that are assigned the value y. We will sometimes denote

Definition 3.3

The probability distribution for a discrete variable can be represented by a formula, a table, or a graph that provides for all

Theorem 3.1

For any discrete probability distribution, the following must be true:

1. 0≤ p(y)≤1 for all y.

2. = 1, where the summation is over all values of with nonzero probability.

Definition 3.4

Let be a discrete random variable with the probability function Then the expected value of is defined to

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Theorem 3.4

Let be a discrete random variable with probability function and be a real-valued function of . Then the expected value of is given by

.

Definition 3.5

If is a random variable with mean the variance of a random variable Y is defined to be the expected value of . That is,

The standard deviation of is the positive square root of

Theorem 3.3

Let be a discrete random variable with probability function and be a constant. Then

Theorem 3.4

Let be a discrete random variable with probability function be a function of, and be a constant. Then

Theorem 3.5

Let be a discrete random variable with probability function and be functions of . Then

Theorem 3.6

Let be a discrete random variable with probability function and mean then

Definition 3.6

A *binomial experiment* possesses the following properties:

1. The experiment consists of a fixed number,, of identical trials.

2. Each trial results in one of two outcomes: success, , or failure, .

3. The probability of success on a single trial is equal to some value p and

remains the same from trial to trial. The probability of a failure is equal to

4. The trials are independent.

5. The random variable of interest is , the number of successes observed

during the trials.

Definition 3.7

A random variable is said to have a *binomial distribution* based on trials with success probability if and only if

and

Theorem 3.7

Let be a binomial random variable based on trials and success probability . Then